Exam. Code : 103201
Subject Code : 1029

B.A./B.Sc. 1st Semester

MATHEMATICS

Paper—II

(Calculus & Trigonometry)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The **Fifth** question may be attempted from any section.

SECTION—A

1. (a) If A_1 and A_2 are two bounded subsets of R, then show that the set :

$$A_1 + A_2 = \{x + y : x \in A_1 \text{ and } y \in A_2\}$$ is bounded. Further if $u_1 = \text{Sup.}A_1$, $u_2 = \text{Sup.}A_2$ then prove that $\text{Sup.}(A_1 + A_2) = u_1 + u_2$.

(b) For what choice of a and b is the function :

$$f(x) = \begin{cases} 3, & \text{if } x \le 2 \\ ax^2 + bx + 1, & \text{if } 2 < x < 3 \\ 7 - ax, & \text{if } x \ge 3 \end{cases}$$

continuous for all x. 5,5

2. (a) Prove that for given $a > 0$ and $b \in R$, there exist a natural number n such that $na > b$.

(b) Define uniform continuity and show that $f(x) = x^2$ is uniformly continuous in [0, 1]. 5,5

SECTION—B

3. (a) If $y = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2 \sinh^{-1}\frac{x}{a}}{2}$, then show that

$$\left(\frac{dy}{dx}\right)^2 - x^2 = a^2.$$

(b) If $y = e^{m \sin^{-1} x}$, then prove that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} = (m^2 + n^2)y_n.$$

Also deduce that $\lim_{x \to 0} \frac{y_{n+2}}{y_n} = m^2 + n^2$. 5,5

4. (a) Prove that :

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}} = 1.$$

(b) State and prove Taylor's theorem with Lagrange's form of remainder after n terms. 5,5

SECTION—C

5. (a) If $\cos(\theta + i\phi) = r(\cos\alpha + i\sin\alpha)$, then prove that

$$\phi = \frac{1}{2}\log\frac{\sin(\theta - \alpha)}{\sin(\theta + \alpha)},\text{ where } \alpha, \theta, \phi, r \in R.$$

(b) Apply De-Moivre's theorem to find an equation whose roots are the nth power of the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$. 5,5

6. (a) If x + iy = cosh(u + iv), then show that

$$\frac{x^2}{\cosh^2 u}+\frac{y^2}{\sinh^2 u}=1 \text{ and } \frac{x^2}{\cos^2 v}+\frac{y^2}{\sin^2 v}=1.$$

(b) Solve $z^7 = 1$ and prove that the sum of nth power of its roots is zero or 7 according as n is not or is a multiple of 7. 5,5

SECTION—D

7. (a) If $i^{\alpha + i\beta} = \alpha + i\beta$, then prove that :

$$\alpha^2 + \beta^2 = e^{-(4n+1)\beta\pi}, n \in Z.$$

(b) Express cos 5θ and sin 5θ in terms of powers of cos θ and sin θ respectively. 5,5

8. (a) Sum to n terms the series :

$\cos\theta \sin\theta + \cos^3\theta \sin 3\theta + \cos^5\theta \sin 5\theta + \text{.........}$ n terms.

(b) Use Gregory series to prove that :

$$1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\text{.........}=\frac{\pi}{2\sqrt{2}}.$$ 5,5